BLOCKING ROTATABLE DESIGNS FOR AGRICULTURAL EXPERIMENTATION

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1. INTRODUCTION

For the exploration of response surfaces in multifactorial experiments Box and Hunter (1957) introduced a class of incomplete factorial designs popularly known as rotatable designs. They defined such designs as follows :

If $x_1, x_2, x_3 \ldots\ldots x_v$ denote v variates with a suitable origin and scale, each representing the levels of a factor and y denotes the responses obtained from N different combinations of the v-variates these combination will be said to form a rotatable design of order d if the surface

$$y=\beta_o+\sum_i \beta_i x_i+\sum_i \sum_j \beta_{ij} x_i x_j+\sum_i \sum_j \sum_k \beta_{ij} k \; x_i x_j x_k+$$

$$\ldots\ldots\ldots\ldots \text{up to power } d$$

can be so fitted with the data collected from these N points that the variance of the response at any point, say, $(x_{10}, x_{20}, x_{30}, \ldots\ldots x_{vo})$ estimated through the response surface is a function of the distance of the point from the origin.

Box and Hunter constructed such designs of orders 2 and 3 through geometrical configurations. Subsequently other methods due to Bose and Draper (1959), Gardiner (1959), Box and Behnken (1960), Das (1961, 1963), Thaker (1961), Das and Narasimhan (1962) and Dey (1969) were also available for the construction of such designs.

These designs have been applied extensively in the fields of industry and chemical engineering. But there does not seem to have been any extensive application of such designs for agricultural field experimentation. The main difficulty for applying such designs in this field is that excepting a few designs almost no work is available in literature where such designs are presented broken into

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blocks of equal size. Though Box and Hunter (1957) have given some second order designs with blocks, the block sizes in all these designs excepting a design in 4 factors are unequal. Box and Behnken (1960), however, gave a design in 4 factors in three blocks of 9 plots each. In experiments with field plots as the experimental units intra block error depends on the block size. It is, therefore, desirable that the blocks of rotatable designs should be of equal size and at the same time not very large.

Through the work of Panse and Khanna (1962), and V.Y. Rao (1957), it seems that the response surfaces in the field of agriculture are likely to be approximated by a second degree surface. Hence, in the present investigation we have restricted the study to second order rotatable designs and have provided some methods for constructing such designs split into blocks of equal but small size.

2. Requirements of Second Order Rotatable Designs

As shown by Box and Hunter (1957), a set of N combinations of the v variates $x_1, x_2, x_3, \ldots\ldots x_v$ with a suitable origin and scale will form a rotatable design if they satisfy the following conditions :

$$(A) : \Sigma\ x_i = 0$$

$$\Sigma\ x_i x_j = 0$$

$$\Sigma\ x_i x_j^2 = 0,\ \Sigma\ x_i^3 = 0,\ \Sigma\ x_i x_j x_k = 0$$

$$\Sigma\ x_i x_j x_k x_l = 0,\ \Sigma\ x_i x_j^3 = 0 \text{ and } \Sigma\ x_i x_j x_k^2 = 0$$

$$(B) : \Sigma\ x_i^2 = \text{constant} = N\lambda_2 \quad \text{(say)}$$

$$\Sigma\ x_i^4 = \text{constant} = 3N\lambda_4 \quad \text{(say)}$$

$$(C) : \Sigma\ x_i^4 = 3\ \Sigma\ x_i^2 x_j^2$$

$$(D) : \frac{\lambda_4}{\lambda_2^2} > \frac{v}{v+2}.$$

Each of the summation in the above relations is over the design points. For example, $\Sigma\ x_i$ denotes summation over all the values of the ith variate x_i in the N points.

Without loss of generality we can take λ_2 to be unity for providing standard scales to the variates.

If the design points are to be distributed into several blocks the following further conditions [Box and Hunter (1957)], should also be satisfied for estimating the constants in the response surface free from block effects :

(*i*) $\Sigma\ x_i^2$ = constant within each block

(*ii*) $\Sigma\ x_i x_j = 0$ within each block.

and (*iii*) $\frac{\Sigma_m\ x_i^2}{\Sigma_l\ x_i^2} = \frac{n_m}{n_l}$ for each pair of blocks.

Here Σ_m denotes summation over the points in the *m*-th block, Σ_l having similar meaning ; n_m and n_l denote the numbers of points in the *m*-th and *l*-th blocks respectively.

3. A Method of Blocking the Central Composite Designs

Box and Hunter (1957) first defined the central composite designs as consisting of n points at the vertices of a cube corresponding to a 2^v factorial arrangement or some suitable fraction of it with co-ordinates ($\pm 1\ \pm 1 \ldots\ldots \pm 1$) together with $n_a = 2k$ 'axial' points with co-ordinates ($\pm\alpha$, 0......0), (0$\pm\alpha$, 0 ......0), ...(0,0..... 0, $\pm\alpha$) and n_0 points at the centre with co-ordinates (00......0).

Adopting slightly different symbolism which has been followed in the present paper, Das and Narasimham (1962) obtained central composite designs in v factors from the set (*i*) (*a a* ..... *a*) and (*ii*) the v combinations (*b* 0... ...0), (0*b*0......0) (00......0*b*) by generating the design points through multiplying by suitable associate combinations. The design points from the first set (*a*, *a*,.........,*a*) are obtained by multiplying it by the combinations of a suitable fraction of the combinations of the fractorial 2^v where each factor in the factorial has the levels 1 and -1. The design points obtainable similarly from the v sets (*b*00 .. 0) (0*b*0......0) (00...... .0*b*) are the same as those given by Box and Hunter wherein the axial point is replaced by *b*. These $2v$ points along with those indicated above constitute the central composite designs.

Instead of taking the design points from the first set (*a a*......*a*) as they are, we may group them into blocks by first grouping combinations of the factorial 2^v into blocks such that no main effect or two factor interaction is confounded for this blocking and then associating (*a a*.........*a*) through multiplication with each of the combinations in such blocks. Now, taking the design points obtainable from the sets (*b* 0 0......0) together with the design points obtained from each of the blocks of the factorial we shall get the blocks of the rotable design. For example, if there be 2^n blocks each of size 2^r of the factorial 2^v or its fraction, which is required for obtaining the design points from the set $(a\ a\ldots\ldots a)$, the final design will have 2^n blocks each of size 2^r+2v. The total number of points in the design will thus be $2^{n+r}+2^{n+1}v$. It can be seen easily that the requirements for blocking are satisfied when blocking is made through this method The value of $\Sigma\, x_i^4$ as defined in section 2 comes out for this design as $2^{n+r}\,a^4+2^{n+1}\,b^4$, and that of

$$\Sigma\, x_i^2\, x_j^2=2^{n+r}\,a^4.$$

Relation C requires that

$$2^{n+r}\,a^4+2^{n+1}\,b^4=3\times 2^{n+r}\,a^4$$

i.e.

$$b^4=2^r\,a^4$$

The method of construction of such designs has been clarified further through the following illustration where a design in 5 factors has been constructed.

For constructing the central composite design in 5 factors we usually take half replicate of the factorial 2^5, the identity group for obtaining the fraction, containing the fourth order interaction. When the fraction is chosen thus, it is not possible to divide the 16 remaining combinations into two blocks of 8 each without confounding a two factor interaction with the blocks. Alternatively all the 32 combinations of the factorial 2^5 may be taken and divided into 4 blocks of 8 each such that no main effects and two factor interactions are confounded. Taking the following as one of the blocks for such confounding

−1	−1	−1	−1	−1
1	−1	−1	1	1
−1	−1	1	−1	1
1	1	−1	−1	1
1	−1	1	1	−1
−1	1	1	1	1
1	1	1	−1	−1
−1	1	−1	1	−1

we can get from it 8 design points by replacing 1 in them by the unknown a.

From the other 3 blocks of the factorial 2^5 three more such groups of design points can be obtained.

Now, taking with each of these 4 groups the 10 design points obtainable from the sets (b 0 0......0) we get four blocks of a central composite design with 18 plots in each block. For this design $b^4 = 8\ a^4$.

4. A Further Method of Blocking the Central Composite Design

It will be seen that the designs obtained in the previous section will normally require a large number of points as the points obtained from the sets (b00......0) are repeated in each block while in the central composite design without blocking these are taken only once. Though such repetition of points provide more degrees of freedom for estimating error variance, it may be desirable in some cases to obtain design with smaller number of points. We have thus given below a further method of construction through which it is possible to obtain designs with smaller number of points as also smaller block size.

This method consists in forming groups of design points from the set (a a........... a) by associating it with contents of each of the 2^n blocks of the factorial 2^v obtained as described in the previous section. Next the points obtained from the sets (b 0 0........ 0) are taken to form another block. We have thus got two types of blocks *viz* (*i*) those obtained from (a a............a) to be called the first type of blocks and (*ii*) that from (b 0......0) to be called the second type of blocks. For the final design we have to include $(2^r - 2^v)$ central points in the block of the second type if $2^r > 2v$ or $(2v - 2^r)$ points in each block of the first type if $2v > 2^r$. This will ensure equality in the size of the two types of blocks. Next the block of second type has to be repeated m times where m has to be determined as described below :

As $\Sigma x^2_i = 2^r a^2$ for each block of the first type and is equal to $2b^2$ for the blocks of the second type. $2^r a^2$ must be equal to $2b^2$ as required for blocking.

Again, Σx^4_i for the whole design is $2^{n+r} a^4 + 2mb^4$ and $\Sigma x^2_i x^2_j = 2^{n+r} a^4$.

Requirement C then gives the equation

$$2^{n+r}\ a^4 + 2mb^4 = 3.2^{n+r}\ a^4$$

$$i.e.,\ m = 2^{n+r}\ \frac{a^4}{b^4}.$$

But from the requirement for blocking, we have

$$b^2 = 2^{r-1}\ a^2$$

Hence m must be equal to 2^{n-r+2}.

The method of blocking the central composite design described above differs from that of Box and Hunter (1957) in that they took the design points obtainable from the sets (b 0.........0), (0 b.........0) (0 0.........0b) only once while in the present investigation these have been repeated a certain number of times so that all the blocks can be made equal in size. By taking these design points only once, they could get a design with equal blocks sizes only for 4 factors.

In this design there will thus be 2^n+m blocks each of size 2^r or $2v$ according as $2^r >$ or $< 2v$.

To continue with the previous example there will now be 4 blocks each of size 2^3 from the set (a a a a a). Thus $n=2$ and $r=3$. Hence the second type of blocks of size 10 has to be repeated m times where $m=2^{n-r+2}=2$.

As $2v>2^r$ in this case, $(2v-2^r)$ *i.e.* 2 central points have to be included in each of the blocks of the first type. The number of points in the design consisting of 6 blocks each of size 10 will be thus 60 while the number of points in the previous design was $4 \times 18 = 72$.

5. Blocking of Second Order Rotatable Designs Obtainable Through B. I. B. Designs

It is known that second order rotatable designs can be obtained by using balanced incomplete block designs with or without equal block sizes (Das and Narasimham 1962, Das 1963). These designs also can be split into blocks of equal size by following a similar procedure as in the case of central composite designs described earlier. One essential requirement for such blocking is that the B. I. B. design or designs used should be resolvable so that the blocks can be divided in several groups, each group containing each of the treatments the same number of times (Box and Behnken 1960 *b*). For the purpose of blocking through the present technique of including the axial points in each block or making separate blocks with them, we shall use two types of B. I. B. designs—one the ordinary resolvable designs with $r \leqslant 3\lambda$ and the other any B. I. B. design with its complementary design.

The first step for blocking the rotatable designs is to put the blocks of the B. I. B. design into several groups such that in each group the treatments are replicated the same number of times. For the resolvable designs formation of such groups does not involve any difficulty. In the case of the complementary designs such groups can be formed by taking any block of a B. I. B. design together with its complementary block containing all the $(v-k)$ treatments which do not occur in the block of the B. I. B. design. Thus, for such design each group will contain two blocks and there will be as many groups as there are blocks in one of the B. I. B. designs.

Next, design points are to be generated from each of these groups of blocks through the procedure described by Das and Narasimham (1962).

In the case of ordinary resolvable designs, the design points obtained from each of the block groups will constitute a block of the rotatable design. In the case of complementary designs if the two blocks in a group be of equal size or the design points obtained from each be equal even though the two blocks are unequal in size, the points obtainable from a block group will constitute a block as in the ordinary resolvable design. When the number of design points obtainable from one of the two blocks of the B I B design in a group is unequal to that obtainable from the other block in the same group, they have to be made equal. This can be done by either repeating the design points obtainable from smaller of the two blocks or dividing the associate combinations from the larger blocks into sub-groups such that no main effect or two factor interaction is confounded. There will thus be as many blocks of the rotatable designs as there are block groups of the B. I. B. design or designs. If the totality of the points in all these blocks satisfy the relation C, they will generate a second order rotatable design with as many blocks and each factor will have three levels.

5.1 *Design Obtained by Repeating the Design Points from the Smaller of the Two Blocks*

Let us take a design in three factors from the following two complementary balanced incomplete block designs :

(*i*)	(1, 2),	(1, 3)	and	(2, 3)
(*ii*)	(1),	(2)	and	(3)

The first design is the usual design while the second one has only one plot per block and hence $\lambda=0$.

With these blocks we form the following three groups of 2 blocks each, one taken from each design :

Groups of blocks

1, 2	1, 3	2, 3
3	2	1

There will be four design points obtainable from each of the block of size 2 while only two points can be obtained from the other block which is of size unity. Hence the block of size unity has to be taken twice so that the number of the points obtainable from it may be the same as obtainable from the other block. The design points in each block of the rotatable design, thus obtained, are shown below ;

Block 1			*Block 2*			*Block 3*		
a	a	0	a	0	a	0	a	a
a	$-a$	0	a	0	$-a$	0	$-a$	$-a$
$-a$	a	0	$-a$	0	a	0	$-a$	a
$-a$	$-a$	0	$-a$	0	$-a$	0	a	$-a$
0	0	a	0	a	0	a	0	0
0	0	$-a$	0	$-a$	0	$-a$	0	0
0	0	a	0	a	0	a	0	0
0	0	$-a$	0	$-a$	0	$-a$	0	0

The design points in any block satisfy all the requirements for blocking. $\Sigma\ x_i^4$ for all these points is $12a^4$ while $\Sigma\ x_i^2\ x^2_j=4a^4$. Hence relation C is automatically satisfied and no further point need be included in any of the blocks. The central points can, however, be included in equal numbers in each of the blocks, when necessary.

5.2 *Design from Resolvable B.I.B. Design (Repetition of the Design Points not Necessary*

For this we shall take the following resolvable B.I.B. design with 4 treatments :

(1, 2) (1, 3) (1, 4) (2, 3) (2, 4) and (3, 4) and group the blocks as shown below :

Group of blocks

(1, 2)	(1, 3)	(2, 3)
(3, 4)	(2, 4)	(1, 4)

The design points obtainable from these groups will give a second order rotatable design in 3 blocks of 8 points each as the relation C is again automatically satisfied by these points. As all the points in this design are equidistant from the centre, it is necessary to take at least one central point in each of the blocks. This design has also been given by Box and Behnken (1960b).

5.3 *Design Obtained by Including Design Points from set (b, 0......0) in the Blocks*

In case the points obtained from b.i.b. designs do not satisfy the relation C the design points obtainable from the set $(b\ 0......0)$ have to be included. These may be included in each of the blocks or in separate blocks. If these points are included in each of the blocks, the size of the block of the rotatable design will be increased by $2v$ points over and above those obtained from a block group of *b.i.b.* design as indicated earlier.

All second order rotatable designs obtainable through resolvable designs with $r \leqslant 3\lambda$ can be split into blocks through this procedure When complementary B.I.B. designs are used, we have

$$\Sigma\, x_i^4 = (r+r')2^{k'}a^4 + 2(r+r')b^4$$

and

$$\Sigma\, x_i^2 x_i^2 = (\lambda+\lambda')2^{k'}a^4.$$

Here r and r' denote the number of replications in a b.i.b. design and its complementary respectively. Similarly λ and λ' are the replications of pairs of treatments in the two designs. $2^{k'}$ denotes the number of design points generated from the greater of the two blocks in a group. For such designs, we have

$$(r+r')2^{k'}a^4 + 2(r+r')b^4 = 3(\lambda+\lambda')2^{k'}a^4$$

i.e.,

$$(r+r')\ b^4 = 3(\lambda+\lambda') - (r+r')2^{k'-1}a^4$$

This equation shows that block of rotatable designs obtainable by using any B.I.B. design with its complementary is always possible as it can be shown that $(\lambda+\lambda')-(r+r')$ is always positive.

We may illustrate this procedure of blocking by taking the following complementary B.I.B. designs with 5 treatments :

(*i*) (1,2) (1,3) (1,4) (1,5) (2,3) (2,4) (2,5) (3,4) (3,5) (4,5)

(*ii*) (3,4,5) (2,4,5) (2,3,5) (2,3,4), (1,4,5) (1,3,5) (1,3,4) (1,2,3) (1,2,4) (1,2,3)

We can now form the groups of blocks by taking one block from the first design and its complementary block from the second design to form a block group. Thus, the blocks (1,2) and (3,4,5) form a group. There will be 10 such groups of two blocks each. As the sizes of the blocks are 2 and 3, we have to repeat the blocks of size 2, so that in all there will be 16 design points in each block of the rotatable design. It will be seen that the 160 points obtained thus from the 10 groups of blocks do not satisfy the relation C.

Hence we have to include 10 design points obtainable from the set $(b,0..........0)$ in each of the 10 blocks obtained earlier. The block size will thus be 26 and each factor will have 5 levels. The value of Σx_i^4 for the whole design is $80a^4+20b^4$ and $\Sigma x_i^2 x_j^2 = 32a^4$.

Hence from the relation C we get $\frac{b^4}{a^4} = 0{\cdot}8$.

5·4. *Design obtained by including design points from set $(b,0 ......0)$ in separate blocks*

In sec. 5 3 the design points from the sets $(b,0..........0)$ are included in each block, the block size will increase considerably. If a reduction in the number of points in each block is desirable we may form separate blocks with these points. As these two types of blocks are unequal they will have to be made equal by including the requisite number of central points in the small block. For satisfying the requirement that Σx_i^2 is constant for each block we may have to repeat the block obtained from $(b,0 ....0)$ a certain number of times, say, m.

For such designs, we have

$$\Sigma x_i^2 = 2^k a^2 \text{ from blocks of } b.i.b. \text{ design}$$

$$= 2b^2 \text{ from blocks of set } (b,0,..........0)$$

Again for the whole design

$$\Sigma x_i^4 = (r+r')\, 2^{k'} a^2 + 2mb^4$$

and

$$\Sigma x_i^2\, x_j^2 = (\lambda+\lambda')\, 2^{k'} a^4.$$

Thus, from relation C,

$$(r+r')\, 2^{k'} a^4 + 2mb^4 = 3(\lambda+\lambda')\, 2^{k'} a^4.$$

Hence

$$m = \frac{3(\lambda+\lambda')-(r+r')}{2^{k'-1}}.$$

If m is a fraction say, $\frac{m_1}{m_2}$ we have to repeat the blocks from the $b.i.b.$ design m_2 times and the blocks obtained $(b,0..........0)$ m_1 times.

For the design given as illustration under Sec. 5·3

$$m = \frac{3(1+3)-(4+6)}{2^{3-1}} = \frac{2}{4} = \frac{1}{2}$$

We have, therefore, to repeat the blocks obtained from the *b.i.b.* designs twice and keep only one block of $(b,0,\ldots\ldots\ldots\ldots 0)$. Thus there will be 21 blocks each of 16 design points.

5 5. *Designs obtained by taking a sub-group of the larger of the two complementary designs and including design points $(b, 0, \ldots\ldots 0)$ either in the block from b.i.b. designs or in separate blocks*

We now consider a situation when a block of rotatable design obtained from a block group of B.I.B. design or designs can be made smaller by first dividing the design points obtainable from each of the blocks in a group into several sub-groups such that in the associate combinations used for obtaining the sub-groups no main effect or two factor interaction is confounded. One sub-group of each of the blocks of B.I.B. designs is put together to form a block of the rotatable design. Through this procedure there will be as many such sub-blocks from a block as there are sub-groups of the larger block. Here also care should be taken to ensure that equal number of design points from each block in the group are included in each block, thus formed. When necessary we may include the $2v$ points of the set $(b,0\ldots\ldots\ldots 0)$ either in each sub-block or in separate blocks. In case these are included in each block the formula for obtaining the value of b^4 in terms of a^4 will remain the same as given in the earlier section. If these points are included in separate blocks we have, if necessary, to make the blocks equal in size by including the requisite number of central points in the smaller block.

For such designs we have :

$$\Sigma x_i^2 = 2^p \times 2^k a^2 \text{ for blocks from } b.i.b. \text{ design}$$

$$= 2b^2 \text{ for blocks from } (b,0\ldots\ldots\ldots\ldots 0)$$

i.e., $b^2 = 2^{p+k-1}\, a^2$.

where 2^p denotes the number of times the design points from the smaller of the blocks of *b.i.b.d.* are repeated in a block of the rotatable design. Again for the whole design

$$\Sigma x_i^4 = (r+r')\, 2^{k'} a^4 + 2mb^4$$

and

$$\Sigma x_i^2 x_j^2 = (\lambda + \lambda')\, 2^{k'} a^4.$$

Hence from relation C

$$(r+r')\, 2^{k'} a^4 + 2mb^4 = 3(\lambda + \lambda')\, 2^{k'} a^4$$

substituting the value of b^4

$$(r+r')\, 2^{k'} a^4 + m 2^{2(p+k)-1} a^4 = 3(\lambda+\lambda')\, 2^{k'} a^4$$

Hence $$m = \frac{3(\lambda+\lambda') - (r+r')}{2^{2(p+k)-k'-1}}.$$

For illustration consider the *b.i.b.* design

$v = b = 7,\ r = k = 3,\ \lambda = 1$ and its complementary

$v=b=7$, $r'=k'=4$, $\lambda'=2$. The grouping of blocking can be done as given below :

124	3567
235	1467
346	1257
457	1236
561	2347
672	1345
713	2456

Let us first consider the blocks obtainable from the block group (1, 2, 4) and (3, 5, 6, 7). As the number of points obtained from (1, 2, 4) is eight while that from other block is 16. The case when we have to repeat the block (1, 2, 4) twice has already been dealt earlier. We, therefore, now divide the 16 points obtainable from (3, 5, 6, 7) into two sub-groups by confounding the four factor interaction. Next, each of these two sub-groups of the design points is taken with the points obtained from the block (1, 2, 4). Thus we have two blocks of rotatable design each of size 16 obtained from each block group of the *b.i.b.* design. There will thus be 7 pairs of such blocks. The two blocks obtained from the first pair are given below :

Block 1							*Block 2*						
a	a	0	a	0	0	0	a	a	0	a	0	0	
a	a	0	$-a$	0	0	0	a	a	0	$-a$	0	0	0
a	$-a$	0	a	0	0	0	a	$-a$	0	a	0	0	0
a	$-a$	0	$-a$	0	0	0	a	$-a$	0	$-a$	0	0	0
$-a$	a	0	a	0	0	0	$-a$	a	0	a	0	0	0
$-a$	a	0	$-a$	0	0	0	$-a$	a	0	$-a$	0	0	0
$-a$	$-a$	0	a	0	0	0	$-a$	$-a$	0	a	0	0	0
$-a$	$-a$	0	$-a$	0	0	0	$-a$	$-a$	0	$-a$	0	0	0
0	0	a	0	a	$-a$	$-a$	0	0	a	0	$-a$	$-a$	$-a$
0	0	a	0	$-a$	a	$-a$	0	0	a	0	a	a	$-a$
0	0	a	0	$-a$	$-a$	a	0	0	a	0	a	$-a$	a
0	0	a	0	a	a	a	0	0	a	0	$-a$	a	a
0	0	$-a$	0	a	a	$-a$	0	0	$-a$	0	a	$-a$	$-a$
0	0	$-a$	0	a	$-a$	a	0	0	$-a$	0	$-a$	a	$-a$
0	0	$-a$	0	$-a$	a	a	0	0	$-a$	0	$-a$	$-a$	a
0	0	$-a$	0	$-a$	$-a$	$-a$	0	0	$-a$	0	a	a	a

Since the relation C is not automatically satisfied we have to take the points from the set (b, 0.........0). We take only the case when the blocks from the points (b, 0.........0) is put in separate blocks. In that case $m=1$.

Hence we have to take a block alongwith the 7 pairs of blocks obtained from *b.i.b.* designs. As the size of this block is 14 while that of the other is 16, two central points may have to be added to this block to make the block size equal. There will thus be 15 blocks each of size 16.

A large number of second order rotatable designs can be split into blocks of equal and small size through the methods described above.

SUMMARY

Two methods of dividing the central composite rotatable designs into blocks of equal size, have been evolved. Methods of blocking second order rotatable designs obtainable through *b.i.b.* designs have also been evolved. For these methods, we may use resolvable *b.i.b.* designs with $r \leqslant 3\lambda$ or *b.i.b.* designs with their complementary *b.i.b.* designs. Through these methods designs with any number of factors can be split into blocks of equal size.

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